

HEDGER RESPONSE TO MULTIPLE GRADES OF
DELIVERY ON FUTURES MARKETS

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I. INTRODUCTION

Futures markets are essentially hedging markets. A successful futures market, therefore, must provide effective hedging instruments for hedgers. Accordingly, contract specifications and regulatory policies are the two most important elements when evaluating the effectiveness of a futures contract. However, due to shifts in the underlying structure of production and trade, contract grades and locations may become less representative of commercial needs, and hence an initially successful futures market may be subject to failure. To solve this problem, new regulatory policies may be proposed; but, the most constructive approach is to adapt the contract specifications to the changing market situation. Such contract specifications include the list of deliverable grades and locations, techniques for financial settlement when non-par deliveries are made, and related terms. (Paul, 1976).

But developing unambiguously "improved" contract specifications is difficult. For example, there is always a tradeoff between the uncertainty of what will be delivered on a futures contract (i.e., the so-called delivery risk (Kamara and Siegel, 1987)), and the likelihood of delivery squeezes and market illiquidity. A contract, such as KCBT (Kansas City Board of Trade) wheat futures before 1940, may allow a single deliverable grade at a single location. The contract specification thus accomodates buyers with the certainty over

what they will receive when taking delivery. Nonetheless, this fragments trading over a large number of futures contracts and thereby reduces overall market liquidity (Garbade and Silber, 1979, 1983a). Moreover, due to the costs of making and taking deliveries, narrow contract specifications enhance the likelihood of squeezes (both short and long squeezes). At maturity, the futures price may diverge from the cash price which greatly reduces hedging effectiveness.

On the other hand, the provision of multiple deliverable grades discourages long hedging due to high delivery risk. In general, the longs can expect to receive the cheapest deliverable grades possible under futures contract (which have long been picturesquely called "skin goods" (Paul, Kahl, and Tomek, 1981)). To correct the bias against longs, a certificate system for delivery may be issued such as in the case of live cattle futures. However, as indicated in Purcell and Hudson (1986), owing to the problems associated with administrative procedures, the results are wider basis and no smaller variability of the basis. It turns out that the certificate system encourages long hedgers at the cost of discouraging shorts. In other words, there is a tradeoff between long and short hedging interests.

Delivery risks are, however, directly related to the structure of premiums/discounts for deliverable grades. If the commercial difference system is adopted such that the premiums/discounts are determined by the differences of cash prices prevailed at the maturity of the futures contract, then there will be no delivery risk (assuming away the costs of

selling unwanted grades and buying desirable grades in the spot markets). But then the provision of multiple grades of delivery will be inoperative and the futures contract reduces to the case of a single delivery (Johnson, 1957) with the problems of squeezes and illiquidity noted earlier. Alternatively, a fixed difference system specifies premiums/discounts in advance. Yet the cash market for different grades does not have fixed discounts or premiums either in absolute or percentage term, because of variations in crop conditions, keeping qualities, demand shift and so on. As a result, there will always be uncertainty as to the valuations of different grades in futures market relative to cash markets. Delivery risk is unavoidable. Thus, Paul, Kahl, and Tomek (1981) concludes that the determination of accurate discounts and premiums is a seemingly intractable problem.

The above arguments have been stated in terms of multiple delivery grades. Once we treat a commodity deliverable at different locations as different "types" of commodities, we can extend the analysis to the case of multiple delivery locations. Clearly, similar tradeoff between short and long hedging interests will occur. Despite these problems, generally a broad-base contract providing for multiple delivery choices is preferred to a narrow-base contract. For example, in the New York cotton contract, after New York City was no longer an important commercial center, the contract was broadened to include deliveries at southern points (Paul, 1976). Also, in 1971, CME (Chicago Merchantile Exchange) replaced Chicago with Omaha as the par delivery point for live cattle futures while

incorporating Guymon of Oklahoma and Peoria of Illinois as nonpar delivery points (Crow, Riley and Purcell, 1972). These are two examples of broadening the list of delivery locations in response to changing market situations.

For the case of multiple delivery grades, examples include KCBT wheat futures, New York coffee futures, and New York potato futures. Specifically, during 1953-1954, the coffee futures was under extensive discussion regarding delivery specifications. It was argued that a broad-base contract should include not only Brazilian coffees but also the many so-called mild growths of other Central and Southern American countries and perhaps African growths as well (Johnson, 1957). In the same fashion, Paul, Kahl, and Tomek (1981) suggested that the contract for potatoes be broadened to permit delivery of all round white potatoes provided they met the same specifications as the Maine variety listed on the contract (also, see Paul (1985)). Finally, KCBT wheat futures provides a more interesting case study. From the beginning of trading in futures at Kansas City (i.e., in 1876), Kansas City was pre-eminently a market for hard Winter wheat. The original KCBT contract allowed delivery of only that class of wheat. In 1940, the contract was changed to allow delivery of soft Winter wheat at the seller's option. The change had little practical effect until the early fifties when prices of soft Winter wheat were depressed relative to the prices of hard Winter wheat. The depression increased to such an extent that Kansas City futures became in effect a soft Winter wheat futures in the spring and summer of 1953. On June 1953, a proposal that the

KCBT wheat futures should revert to strictly hard Winter wheat futures was made and subsequently rejected. A second proposal that a separate hard wheat futures contract be introduced for use as an alternative to the existing contract was, however, approved in October 1953 (Working, 1954).

II. HYPOTHETICAL FRAMEWORK

Clearly, if we assume no fixed cost in transacting in a market and no cost to formulate a probability distribution for the futures price, extending the list of futures markets will not decrease the expected utility of hedgers (Anderson and Danthine, 1981). However, more futures markets lead to the fact that fewer firms will participate in any particular market, which in turn implies a lower liquidity level (Ward and Behr, 1983) and higher transaction costs. Consequently hedging effectiveness may actually decrease. This issue will not be discussed here; instead, we concentrate on evaluating the (dis)advantage of a broad-base contract against a narrow-base contract.

In the following, we adopt Johnson's hypothetical framework (Johnson, 1957). Specifically, we compare two contracts: a narrow-base contract that allows delivery of one and only one grade W (say, no. 2 red hard wheat), and a broad-base contract permitting the delivery of either W or an alternative grade Z (say, no. 2 red soft wheat). Assuming that delivery on the hypothetical contracts would not itself affect the spot prices, and disregarding the possibility of squeezes in comparing the two contracts, Johnson argued that there is no reason on a priori

grounds to presume that a narrow-base contract would be a less effective instrument than a broad-base contract; in fact, under certain circumstances that are likely to prevail, the opposite may as well be true. His arguments are based on the uncertainty of predicting the basis grade associated with a broad-base contract, since at times the contract would be priced in terms of the expected delivery of Z while at other times in terms of the expected delivery of W. Thus, although the broad-base contract may increase the hedging effectiveness for Z, the gain would probably be more than offset by the loss of effectiveness for W.

We attempt to formalize Johnson's analysis in this paper.¹ To begin, we consider a representative short hedger (an elevator operator) who stores both grades of wheat. When the futures contract allows only the delivery of W, he has a perfect hedge on W (i.e., the risk will be eliminated by taking an equal but opposite futures position against the cash position) because the futures price and the spot price of the grade are equal at the maturity date. On the other hand, as long as the spot price of Z is not perfectly correlated with the spot price of W, he has to cross hedge Z. As a result, the risk he faces pertains to Z. Now, given a broad-base contract, both grades are deliverable; at maturity of the futures contract, the futures price will be set at the minimum of the two spot prices to eliminate arbitrage opportunities.² In fact, short hedgers maximize profits by delivering the grade which is cheapest in the cash market (Hoffman, 1932). The perfect hedge on W is thus unavailable; instead, operations in both grades will incur risks.

Although the risk level of W increases in the broad-base

contract (relative to the narrow-base contract), the risk level of Z may be reduced due to increasing correlation between the futures price and the spot price of Z (Note that there is always some probability that the contract is priced in terms of the expected delivery of Z whereas the narrow-base contract is always priced in terms of W). In other words, the broad-base contract creates a tradeoff between the risk levels of the two grades. Consequently, to compare the two contracts, we have to compare the choice of two lotteries by hedgers. The first lottery, corresponding to the narrow-base contract, incurs zero risk for W and a great risk for Z; the second lottery, corresponding to the broad-base contract, incurs equal, positive risk levels for both grades but the risk is less than that for Z in the first lottery. In general, we would then expect ambiguous results concerning changes in welfare level and futures commitment of hedgers.

Within a mean-variance framework,³ and assuming that martingale equilibria always prevail, we characterize the conditions under which the broad-base contract may lead to a welfare loss and/or decreasing futures position for a representative hedger. In other words, we not only verify the ambiguity results argued by Johnson (1957), but also identify the environments such that short hedgers may actually prefer a narrow-base contract to a broad-base contract. (Note that long hedgers always prefer the former contract; hence Johnson's arguments are relevant to short hedgers only).

In fact, assuming an arbitrary bivariate density function for the two spot prices, we show that the cash position of W is

always smaller in the case of a broad-base contract while the cash position of Z may be smaller or larger. If we adopt a bivariate normal density function such as Khoury and Martel (1985), and Kamara and Siegel (1987), then the cash position of Z always increases when comparing the broad-base contract to the narrow-base contract. Finally, the greater is the correlation between the two spot prices, the higher is the expected utility derived by short hedgers from the broad-base contract; and hence the contract is more desirable as compared to the narrow-base contract the greater is the correlation of the spot prices. Undesirable broad-base contracts are, in general, associated with low storage costs and/or large risk aversion coefficients.

III. THE CASE OF A NARROW-BASE CONTRACT

We assume a two-period world where $t = 0$ is the present time and $t = 1$ is the maturity date for both hypothetical contracts. An elevator operator buys the two grades at $t = 0$, stores and then sells at $t = 1$. Let P_{w0} and P_{w1} (resp. P_{z0} and P_{z1}) denote the spot prices of W (resp. Z) at $t = 0$ and $t = 1$, respectively. Prices are exogenously determined such that P_{w0} and P_{z0} are known to the hedger, but P_{w1} and P_{z1} are random variables. Following the empirical finding of Labys and Granger (1970), we assume that spot prices follow a martingale process such that current spot prices are unbiased predictors of future spot prices:⁴ $E(P_{w1}) = P_{w0}$ and $E(P_{z1}) = P_{z0}$, where $E(\cdot)$ is the expectation operator conditional on information available at $t = 0$. Furthermore, we assume P_{w1} and P_{z1} have the same marginal

probability density function.⁵ Thus, upon appropriately choosing the measurement units, we may assume $\text{var}(P_{w1}) = \text{var}(P_{z1}) = 1$ and $\text{cov}(P_{w1}, P_{z1}) = p$, $0 < p < 1$. The storage cost for each grade is specified as: $(c_i/2)(S_i - d_i)^2$, where $c_i > 0$ is the cost factor. S_i is the cash position of grade i , d_i is the cost minimizing inventory level for grade i , $\forall i = W, Z$. Due to convenience yield considerations, d_i is assumed to be positive (Lien, 1986). To isolate the effects of different hypothetical contracts from cost factors, we assume $c_i = c$, $d_i = d$, $\forall i = W, Z$; thus, the two grades have the same storage cost function.

By participating in the futures market, the elevator operator would reduce his risks. More specifically, he may hedge his cash position by purchasing or selling V futures contracts (it is a purchase when $V > 0$; a sale when $V < 0$). Let P_{f0} and P_{f1} denote the futures prices at $t = 0$ and $t = 1$, respectively. We assume a martingale equilibrium characterizes the futures market⁶ such that $E(P_{f1}) = P_{f0}$. Since in the narrow-base contract only W is deliverable, the contract is priced in terms of the expected delivery of W ; consequently, at the maturity $P_{f1} = P_{w1}$ which implies $\text{var}(P_{f1}) = \text{var}(P_{w1}) = 1$, $\text{cov}(P_{f1}, P_{w1}) = 1$, and $\text{cov}(P_{f1}, P_{z1}) = p$.

Given the above structure, we adopt a mean-variance approach with λ being the risk aversion coefficient ($\lambda > 0$) such that a more risk averse hedger has a larger λ . To maximize his expected utility (the problem formulation is relegated to Appendix I), the hedger chooses his optimal cash and futures positions as follows:

$$S_w^* = d; S_z^* = ed/(1 - p^2 + e); V^* = -d - edp, \quad (1)$$

where $e = c/\lambda$. Note that the hedger will choose the cost minimizing inventory level (i.e., $S_w^* = d$) for W since the risk in the cash position of W could be totally eliminated by an equal but opposite futures position, which implies the inventory cost is the only concern to him. However, the optimal cash position of Z is smaller than the cost minimizing level because there is always some residual risk for cross hedging Z; whereas the cross hedging ratio is simply determined by p, the covariance between the cash price of Z and the futures price given that $\text{var}(P_{f1}) = 1$. Furthermore, since the two storage cost functions are separate and additive, the optimal futures position consists of two parts: (i) $V_w^* = -d$ which is a perfect hedge against W; and (ii) $V_z^* = -edp/(1-p^2+e)$ which is the cross hedge against Z such that $V^* = V_w^* + V_z^*$. The expected utility level corresponding to the optimal cash and futures positions is

$$u^* = -cd^2(1-p^2)/2(1-p^2+e), \quad (2)$$

which is the highest expected utility level the hedger may attain.

Table 1 summarizes the directional effects of changes in the four parameters (i.e., d, p, c, and λ) on S_w^* , S_z^* , $-V^*$ and u^* . [The magnitudes of these effects are described in Appendix II]. Specifically, when the cost minimizing inventory level (d) increases, the hedger will respond by increasing the cash positions of both grades in order to reduce inventory cost. Since the optimal hedging ratios are fixed [i.e., the ratio is 1 for hedging W and p for cross hedging Z], the futures position

$(-V^*)$ also increases where $-V^*$ is the optimum number of contracts the hedger will sell. As a result, the maximum expected utility level (u^*) decreases because more risk will be incurred when the cash position of Z increases as Z can only be cross hedged in a narrow-base contract. When the cost factor c increases, any deviation from the cost minimizing inventory level incurs larger inventory cost. Thus, the hedger will increase his cash position of Z toward the cost minimizing level (note that the optimal cash position of W is always set at this level, hence it is independent of changes in c , p or λ). The result is then similar to the above case. Furthermore, if the two spot prices have a larger correlation (i.e., p increases), then the narrow-base contract becomes a more effective cross hedging instrument for Z since the futures price and the cash price of Z are more likely to move together. In this case, by participating in the futures market, the residual risk after cross hedging is smaller. The consideration of inventory cost outweighs that of the risk level, which leads the hedger to increase his cash position of Z toward the cost minimizing level. The optimal futures position will also increase due to a larger cash position and a larger cross hedging ratio (reflecting the increasing effectiveness of cross hedging). As a consequence, the inventory cost and the risk level both are reduced, and the hedger is able to attain a higher expected utility level. Finally, if the hedger is more risk averse (i.e., λ increases), any price uncertainty becomes more detrimental; correspondingly, he will be less active in both cash and futures markets. The optimal cash position of Z then

decreases. Given a fixed cross hedging ratio, the optimal futures position also decreases. The hedger thus attains a lower expected utility level.

IV. THE CASE OF A BROAD-BASE CONTRACT

In the broad-base contract, Z is also deliverable. To eliminate arbitrage opportunities, at the maturity, we must have $P_{f1} = \min(P_{w1}, P_{z1})$. Following Johnson (1957), we will disregard the possibility of squeezes when comparing the two contracts. Furthermore, we assume the spot markets are perfectly competitive such that hedgers' decisions (including making delivery) do not affect the equilibrium spot price levels. Thus, the equilibrium spot prices for the two grades are maintained at P_{w0} and P_{z0} (resp., P_{w1} and P_{z1}) when $t = 0$ (resp., $t = 1$). Also, $E(P_{w1}) = P_{w0}$ and $E(P_{z1}) = P_{z0}$ remain valid. We further assume that the futures market under the broad-base contract specification is established at a martingale equilibrium such that $P_{f0} = E(P_{f1}) = E[\min(P_{w1}, P_{z1})]$. In other words, we assume martingale equilibria always prevail for either the narrow or broad based contract. Nonetheless, the equilibrium futures price at $t = 0$ may be different across the two contracts. In general, the specification of multiple grades of delivery is expected to depress the futures price level prior to the delivery date (Garbade and Silber, 1983a; Gay and Manaster, 1984; Paul, 1985). Thus, we may expect P_{f0} to be smaller in the broad-base contract than in the case of the narrow-base contract. The reason is simply that the delivery risk (which reduces hedging effectiveness) will be taken into account when pricing the

contract at $t = 0$.

Now, let β denote $\text{cov}(P_{w1}, P_{f1})$ and let γ denote $\text{var}(P_{f1})$. Since P_{w1} and P_{z1} are assumed to have the same marginal probability density function, we have $\text{cov}(P_{z1}, P_{f1}) = \text{cov}(P_{w1}, P_{f1}) = \beta$. Upon following similar mean-variance consideration, the optimal cash and futures positions for the hedger are:⁷

$$\hat{S}_w = \hat{S}_z = ed\gamma / [\gamma(1+e+p) - 2\beta^2] > 0; \quad (3)$$

$$\hat{V} = -2ed\beta / [\gamma(1+e+p) - 2\beta^2] < 0. \quad (4)$$

The maximum attainable expected utility level (i.e., the expected utility corresponding to optimal cash and futures positions) is then

$$\hat{u} = -cd^2[\gamma(1+p) - 2\beta^2] / [\gamma(1+e+p) - 2\beta^2]^2 < 0. \quad (5)$$

In other words, it is always better for the elevator operator to participate in the futures market by selling broad-base contracts (i.e., $-\hat{V} > 0$). On the other hand, because of the delivery risk, there is residual risk that cannot be hedged; the hedger's cash positions for both grades are, therefore, smaller than the cost minimizing inventory level (i.e., $\hat{S}_w < d$, $\hat{S}_z < d$).

As shown in Table 2, the directional effects of changes in the four parameters upon the optimal cash and futures positions, and upon the maximum expected utility level are similar to those presented in Table 1; except now the cash positions of the two grades receive the same impacts. The analysis is also similar to the case of the narrow-base contract. For example, when the cost minimizing inventory level increases (i.e., d increases), the

hedger will respond by increasing his cash positions in order to reduce inventory cost. Since the hedging ratio is fixed at $\text{cov}(P_{w1}, P_{f1})/\text{var}(P_{f1}) = \beta/\gamma$ for either grade, the futures position also increases. As a result, the expected utility level decreases due to the fact that more risk will be incurred through a greater transaction volume in the futures market. The effects of changes in the correlation between the two spot prices, however, are ambiguous because of the dependence of β and γ upon p (for details, see Appendix II). If we follow Khoury and Martel (1985) and assume that $(P_{w1}, P_{z1})'$ has a bivariate normal density function, then it can be shown that $\beta = (1+p)/2$, and $\gamma = 1 - (1-p)/\pi$ (Kamara and Siegel, 1987). The relationship between β and p , and γ and p are then explicitly established. Upon applying these two properties, we can show that the effects of changes in the correlation also parallel to the cases described in Table 1. That is, given a bivariate normal density function for the two spot prices, the cash positions, the futures position, and the maximum expected utility level all increase with increasing correlation between the two spot prices. As a result, the directional effects of changes in the parameters are the same for both narrow-base and broad-base contracts.

V. COMPARISONS OF THE TWO HYPOTHETICAL CONTRACTS

We are now in the position to compare the two contracts. First, it can easily be shown that the cash position for W is smaller in the case of the broad-base contract as compared to the narrow-base contract (i.e., $\hat{S}_W < S_W^*$; a similar result is established in Garbade and Silber (1983b)). Essentially, given

the narrow-base contract, the hedger will choose his optimal cash position for W at the cost minimizing level because the contract allows a perfect hedge. On the other hand, the delivery risk incorporated in the broad-base contract implies there is always some residual, unhedgeable risk. The hedger, therefore, chooses his optimal cash position at a point less than the cost minimizing level. Nonetheless, the relationship between the two optimal cash positions for Z is ambiguous. More precisely,

$$\hat{S}_Z > S_Z^* \text{ if and only if } 2\beta^2 > p(1+p)\gamma. \quad (6)$$

In other words, although the correlation between the spot price of Z and the futures price increases in the broad-base contract, it is not sufficient to ensure an increasing cash position for Z as compared to the case of the narrow-base contract. In fact, given the broad-base contract, the hedger will choose equal cash positions for W and Z in order to obtain the maximum expected utility. (The property is obvious since W and Z are symmetric in every aspect, i.e., they have the same probability density function for their prices, the same storage cost function). Hence a larger cash position for Z implies a larger position for W; the former incurs less risk in the broad-base contract, but the latter incurs more risk. The net change in risk may be larger or smaller, which in turn determines whether or not a larger cash position of Z is preferred. Ambiguous results are thus created. If we assume a bivariate normal density function for $(P_{W1}, P_{Z1})'$, then it can be shown that $2\beta^2$ is always greater than $p(1+p)\gamma$, and hence $\hat{S}_Z > S_Z^*$. That is, the cash position of

Z is always larger in the case of the broad-base contract, which is consistent with the popular belief that a broad-base contract favors cross hedgers.

As the two grades are symmetric, we may compare the total cash position for the two contracts, i.e., $\hat{S}_W + \hat{S}_Z$ and $S_W^* + S_Z^*$. In general, we would expect an ambiguous result because the decrease in the cash position of W in the broad-base contract may be offset by the increase of the cash position of Z. More specifically,

$$\hat{S}_W + \hat{S}_Z \gtrless S_W^* + S_Z^* \text{ if and only if } 2\beta^2(2e+1-p^2) \gtrless \gamma(1+p^2)(1+e-p). \quad (7)$$

Provided that $4\beta^2 > \gamma(1+p^2)$ (which holds for any bivariate normal density function), the total cash position will be larger in the case of the broad-base contract whenever e is sufficiently large. Since $e = c/\lambda$, we may conclude that the total cash position is larger when the storage cash factor (c) is sufficiently large, and/or the risk aversion coefficient (λ) is sufficiently small.

Intuitively, when the cost factor is large, the narrow-base contract leads to a great storage cost for Z because the grade can only be cross hedged and the resulting cash position is smaller than the cost minimizing level. Given the broad-base contract, the increasing hedging effectiveness implies a higher storage level for Z; the storage cost is then smaller. On the other hand, a small risk aversion coefficient ensures that the hedger will not worry too much about the increasing risk. Thus, the storage cost is reduced at the expense of a small loss associated with increasing risk, which is certainly worthwhile.

As a result, the total cash position for the broad-base contract is larger than that for the narrow-base contract when e is sufficiently large. Table 3 shows that, if the two spot prices have a bivariate normal density function with p being the correlation coefficient, then the total cash position in the case of the broad-base contract is always greater whenever $p \geq 0.2$. When $p = 0.1$, the condition that $e > 0.101$ is sufficient to maintain the above conclusion.

Similarly, when we compare futures positions for the two contracts, ambiguous results are produced. In fact, $(-\hat{V}) \stackrel{?}{\geq} (-V^*)$ if and only if

$$2e\beta(1-p^2+e)+2\beta^2(1+p)(1-p+e)-\gamma(1+p)(1-p+e)(1+p+e) \stackrel{?}{\geq} 0. \quad (8)$$

To proceed further, again we assume a bivariate normal density function for $(P_{w1}, P_{z1})'$. The result is shown in Table 3. More specifically, when e is sufficiently large, the futures position for the broad-base contract is greater than that for the narrow-base contract. This is not surprising because futures contracts are hedging instruments; therefore, the futures position should reflect the total cash position. As stated above, when e is sufficiently large, the total cash position for the broad-base contract is larger than the case of the narrow-base contract, hence the futures position is also larger. On the other hand, when the correlation between the two spot prices increases (i.e., p increases), it is more likely for the broad-base contract to establish a larger futures position (i.e., as shown in the table, the lower bound of e decreases as p increases). In any case, the

hedger will be more active in the broad-base contract transactions whenever the storage cost factor (c) is very large or when he is very risk averse.

Finally, if we compare the expected utility levels, then $u > u^*$ if and only if

$$2\beta^2(1-p^2+2e) > \frac{\gamma}{\lambda} \gamma(1+p)(1-p)(1+p+e). \quad (9)$$

Upon adopting a bivariate normal density function, we have similar results. That is, the hedger will derive more expected utility from the broad-base contract when the storage cost factor is sufficient large or/and the risk aversion coefficient is sufficiently low (see Table 3). Also, as the correlation between the two spot prices increases, it is more likely that the broad-base contract generates welfare gains for the hedger in comparison to the narrow-base contract. Therefore, it is more likely that the hedger will prefer the broad-base contract.

In sum, the above analysis indicates there are some cases that the hedger may prefer the broad-base contract, also some cases that the narrow-base contract is preferred. The key factors are the correlation between spot prices, storage cost factor, and the risk aversion coefficient. Thus, when choosing the broad-base contract against the narrow-base contract for a commodity, it is important to identify quantitative values for these factors. Before concluding the paper, note that there are instances such that the total cash position is larger in the case of the broad-base contract yet both futures position and the expected utility are smaller (for example, $p = 0.8$ and $e = 0.01$). It is also possible that the futures position is larger but the

expected utility is smaller (say, $p = 0.3$, $e = 0.5$), or that the futures position is smaller but the expected utility is larger (say, $p = 0.7$, $e = 0.1$) when comparing the broad-base contract to the narrow-base contract.

VI. CONCLUDING REMARKS

In this paper, we provide a more rigorous analysis of contract comparisons based on Johnson's arguments. The results indicate that, for some instances, the short hedger may prefer the narrow-base contract to the broad-base contract. The welfare analysis is with respect to an individual hedger and is not a market level analysis. In fact, when evaluating futures market performance, the most important criteria may be the stability (or variability) of the market basis (Purcell and Hudson, 1986). In our case, since the long hedger always prefers the narrow-base contract, once the short hedger expresses the same preference, it is reasonable to argue that the narrow-base contract should be a better hedging instrument for the market. This situation is generally ignored in the literature; however, we explicitly establish its possibility here. On the other hand, if the short hedger prefers the broad-base contract, we have conflicting interests between the short and long hedgers. Market level criteria then must be applied to the choice of contract specifications. The topic is left for future research.

Secondly, when comparing the two contracts, we disregard the possibility of market manipulation. Nonetheless, the broad-base contract is often initiated to reduce the extent of delivery

squeezes. This adds an additional incentive for the short hedger to favor the broad-base contract. Consequently, our results may be subject to revision. Finally, although we only compare the two hypothetical contracts in this paper, the approach applies to the general case as well. A general framework incorporating the possibility of market manipulation and the market-level criteria will help clear up the issues related to contract specifications.

FOOTNOTES

1. The approach adopted here is similar to that of Anderson and Danthine (1981). However, they consider the effects of adding a new futures contract for Z while we consider the case of incorporating Z as an alternative deliverable grade into the existing contract. In other words, they compare the case of a single futures contract to that of two futures contracts; in our model, a narrow-base contract is compared to a broad-base contract.

2. In a competitive economy, spot prices of the two grades will be simultaneously determined by market clearing. Given that the two grades are not perfect substitutes for each other, generally the two prices will differ from each other. If the futures price exceeds the price of the cheapest deliverable grade, riskless arbitrage opportunities exist by selling futures and buying the cheapest grade to deliver against the contract. If, on the contrary, the futures price is short of the price of the cheapest grade, then buying futures and taking delivery again leads to positive profits.

3. The mean-variance framework is equivalent to expected utility maximization when net revenues are normally distributed and the hedgers have exponential utility functions. Similar results could be obtained by allowing a general expected utility formulation, expanding the first order conditions by a Taylor series, and arguing that the first and second moments of the distribution dominate the higher-order terms

and are good proxies to be used for real world decision making (Anderson and Danthine, 1981; Newbery, 1987).

4. The assumptions are made for mathematical simplicity. For the general case such that $E(P_{w1}) \neq P_{w0}$ or/and $E(P_{z1}) \neq P_{z0}$, our results are retained qualitatively. Also, note that the minimum inventory cost is attained at some nonzero level, hence there is an incentive for hedgers to carry stocks despite the fact that $E(P_{w1}) = P_{w0}$ and $E(P_{z1}) = P_{z0}$. Obviously, when the minimum inventory cost is attained at zero, the two assumptions lead to zero storage level for both grades.
5. This is a strong assumption; at least, it implies that the two spot prices have the same mean and the same variance. Kamara and Siegel (1987) argued that the two prices are actually prices of different grades of the same commodity, therefore they should have the same variance. The conjecture is empirically supported using the sample data from January 1970 to March 1981. On the other hand, if the fixed difference is accurate, then the two spot prices should have the same mean (upon taking into account of the delivery costs for either grade) since both are deliverable at par given the contract specifications.
6. The assumption that the futures market exhibits a martingale equilibrium is a strong one. In fact, a contango or backwardation equilibrium may exist as well. The approaches described in the paper also apply to the latter cases.

However, to specify the difference between $E(P_{f1})$ and P_{f0} , the behavior of cash markets has to be introduced into our framework. The problem then becomes much more complicated.

7. From Appendix I, we have the variance-covariance matrix for $(P_{w1}, P_{z1}, P_{f1})'$ in the case of the broad-base contract.

Since the matrix is positive definite, its determinant must be positive, which implies $\gamma(1+p) > 2\hat{\epsilon}^2$. Also, $\gamma(1+e+p) > 2\hat{\epsilon}^2$. Upon applying these properties to equations (3)-(5), obviously $\hat{S}_w = \hat{S}_z > 0$, $\hat{V} < 0$, and $\hat{u} < 0$.

APPENDIX I

Upon adopting a mean-variance approach, the expected utility for the hedger may be written as:

$$E[(P_{w1}-P_{w0})S_w + (P_{z1}-P_{z0})S_z + (P_{f1}-P_{f0})V] - (c/2)(S_w-d)^2 - (c/2)(S_z-d)^2 - (\lambda/2)\text{var}[(P_{w1}-P_{w0})S_w + (P_{z1}-P_{z0})S_z + (P_{f1}-P_{f0})V],$$

where λ is the risk aversion coefficient and $\text{var}(\cdot)$ is the variance operator. Since P_{i0} is known at $t = 0$, also $E(P_{i1}) = P_{i0}$, $\forall i = w, z, f$, the above expression reduces to:

$$-(c/2)(S_w-d)^2 - (c/2)(S_z-d)^2 - (\lambda/2)(S_w, S_z, V)\Sigma(S_w, S_z, V)',$$

where Σ is the variance-covariance matrix of $(P_{w1}, P_{z1}, P_{f1})'$, and " ' " is the matrix transpose operator. For the narrow-base contract, we have

$$\Sigma = \begin{bmatrix} 1 & p & 1 \\ p & 1 & p \\ 1 & p & 1 \end{bmatrix} .$$

In the case of the broad-base contract,

$$\Sigma = \begin{bmatrix} 1 & p & \beta \\ p & 1 & \beta \\ \beta & \beta & \gamma \end{bmatrix} ,$$

where $\beta = \text{cov}(P_{w1}, P_{f1}) = \text{cov}(P_{z1}, P_{f1})$ and $\gamma = \text{var}(P_{f1})$.

From the point of view of hedging theory, we may define $h_w = -V_w/S_w$ and $h_z = -V_z/S_z$ such that h_w (resp. h_z) is the hedging ratio for W (resp. Z) and $V_w + V_z = V$. Consequently, the

variance term may be rewritten as follows:

$$\begin{aligned}
 & \text{var}[(P_{w1}-P_{w0})S_w + (P_{z1}-P_{z0})S_z + (P_{f1}-P_{f0})V] \\
 = & \text{var}[P_{w1}S_w + P_{z1}S_z + P_{f1}V] \\
 = & \text{var}[P_{w1}S_w + P_{z1}S_z + P_{f1}(V_w+V_z)] \\
 = & \text{var}[S_w(P_{w1}-P_{f1}h_w) + S_z(P_{z1}-P_{f1}h_z)] \\
 = & \text{var}[S_w(B_{w1}+(1-h_w)P_{f1}) + S_z(B_{z1}+(1-h_z)P_{f1})], \tag{A1}
 \end{aligned}$$

where $B_{w1} = P_{w1} - P_{f1}$ is the basis for W at $t = 1$; $B_{z1} = P_{z1} - P_{f1}$ is the basis for Z at $t = 1$. That is, the hedger's variance contains the basis variances for both grades and a futures price variance. In fact, equation (A1) may also be written as:

$$\begin{aligned}
 & \text{var}[S_w((B_{w1}-B_{w0}) + (1-h_w)(P_{f1}-P_{f0})) + S_z((B_{z1}-B_{z0}) + \\
 & (1-h_z)(P_{f1}-P_{f0}))], \tag{A2}
 \end{aligned}$$

where $B_{w0} = P_{w0} - P_{f0}$ is the basis of W at $t = 0$; $B_{z0} = P_{z0} - P_{f0}$ is the basis of Z at $t = 0$. Clearly, both basis risk variances are components of the hedger's variance.

APPENDIX II

To investigate the effects of changes in parameters within the narrow-base contract, we take partial derivatives with respect to d , p , c , and λ in equations (1)-(2) to derive the following properties:

$$(i) \quad \frac{\partial S_w^*}{\partial d} = 1; \quad \frac{\partial S_w^*}{\partial p} = \frac{\partial S_w^*}{\partial c} = \frac{\partial S_w^*}{\partial \lambda} = 0;$$

$$(ii) \quad \frac{\partial S_z^*}{\partial d} = e/k > 0; \quad \frac{\partial S_z^*}{\partial p} = 2edp/k > 0;$$

$$\frac{\partial S_z^*}{\partial c} = d(1-p^2)/\lambda k > 0; \quad \frac{\partial S_z^*}{\partial \lambda} = -cd(1-p^2)/\lambda^2 k < 0;$$

$$(iii) \quad \frac{\partial (-V^*)}{\partial d} = 1+(ep/k) > 0; \quad \frac{\partial (-V^*)}{\partial p} = ed(1+e+p^2)/k > 0;$$

$$\frac{\partial (-V^*)}{\partial c} = pd(1-p^2)/\lambda k > 0; \quad \frac{\partial (-V^*)}{\partial \lambda} = -pcd(1-p^2)/\lambda^2 k < 0;$$

$$(iv) \quad \frac{\partial u^*}{\partial d} = -cd(1-p^2)/k < 0; \quad \frac{\partial u^*}{\partial p} = pcd^2e/k^2 > 0;$$

$$\frac{\partial u^*}{\partial c} = -d^2(1-p^2)^2/2k^2 < 0; \quad \frac{\partial u^*}{\partial \lambda} = -e^2d^2(1-p^2)/2k^2 < 0,$$

where $k = 1 - p^2 + e > 0$.

In the case of the broad-base contract, we take partial derivatives with respect to d , p , c , and λ in equations (3)-(5). The results are:

$$(i) \quad \frac{\partial \hat{S}_w}{\partial d} = \frac{\partial \hat{S}_z}{\partial d} = e\gamma/m > 0;$$

$$\frac{\partial \hat{S}_w}{\partial p} = \frac{\partial \hat{S}_z}{\partial p} = -ed(\gamma^2 + 2\beta^2(\partial\gamma/\partial p) - 4\beta\gamma(\partial\beta/\partial p))/m^2 \stackrel{?}{>} 0;$$

$$\frac{\partial \hat{S}_w}{\partial c} = \frac{\partial \hat{S}_z}{\partial c} = d\gamma n/\lambda m^2 > 0; \quad \frac{\partial \hat{S}_w}{\partial \lambda} = \frac{\partial \hat{S}_z}{\partial \lambda} = -cd\gamma n/\lambda^2 m^2 < 0;$$

$$(ii) \quad \frac{\partial(-\hat{V})}{\partial d} = 2e\beta/m > 0;$$

$$\frac{\partial(-\hat{V})}{\partial p} = 2ed(\beta\gamma - (m+4\beta^2)(\partial\beta/\partial p) + \beta(1+e+p)(\partial\gamma/\partial p))/m^2 \stackrel{?}{>} 0;$$

$$\frac{\partial(-\hat{V})}{\partial c} = (2\beta/\gamma)(\partial \hat{S}_w/\partial c) > 0; \quad \frac{\partial(-\hat{V})}{\partial \lambda} = (2\beta/\gamma)(\partial \hat{S}_w/\partial \lambda) < 0;$$

$$(iii) \quad \frac{\partial \hat{u}}{\partial d} = -2cdn/m^2 < 0;$$

$$\frac{\partial \hat{u}}{\partial p} = -cd^2(\gamma^2 e + 2\beta^2 e(\partial\gamma/\partial p) - 4\beta e(\partial\beta/\partial p))/m^2 \stackrel{?}{>} 0;$$

$$\frac{\partial \hat{u}}{\partial c} = -n^2 d^2/m^2 < 0; \quad \frac{\partial \hat{u}}{\partial \lambda} = -\gamma e^2 d^2 n/m^2 < 0,$$

where $m = \gamma(1+e+p) - 2\beta^2 > 0$, $n = \gamma(1+p) - 2\beta^2 > 0$.

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TABLE 1. DIRECTIONAL EFFECTS OF CHANGES IN PARAMETERS
(THE CASE OF THE NARROW-BASE CONTRACT)

<u>increase</u> <u>in</u>	<u>change in the</u> <u>cash position</u> <u>of W</u>	<u>change in the</u> <u>cash position</u> <u>of Z</u>	<u>change in the</u> <u>futures position</u>	<u>change in the</u> <u>expected</u> <u>utility level</u>
cost mini- mizing inventory level	+	+	+	-
correlation of spot prices	0	+	+	+
storage cost factor	0	+	+	-
risk aver- sion coeffi- cient	0	-	-	-

TABLE 2. DIRECTIONAL EFFECTS OF CHANGES IN PARAMETERS
(THE CASE OF THE BROAD-BASE CONTRACT)

<u>increase</u> <u>in</u>	<u>change in the</u> <u>cash position</u> <u>of W</u>	<u>change in the</u> <u>cash position</u> <u>of Z</u>	<u>change in the</u> <u>futures position</u>	<u>change in the</u> <u>expected</u> <u>utility level</u>
cost mini- mizing inventory level	+	+	+	-
correlation of spot prices	?	?	?	?
storage cost factor	+	+	+	-
risk aver- sion coeffi- cient	-	-	-	-

TABLE 3. THE CASE OF BIVARIATE NORMAL DENSITIES

correlation of spot prices	$A(e)^1$	$B(e)^2$	$C(e)^3$
0.1	$e > 0.101$	$e > 0.513$	none
0.2	$e > 0$	$e > 0.456$	$e > 36.478$
0.3	$e > 0$	$e > 0.399$	$e > 1.091$
0.4	$e > 0$	$e > 0.323$	$e > 0.426$
0.5	$e > 0$	$e > 0.285$	$e > 0.207$
0.6	$e > 0$	$e > 0.228$	$e > 0.103$
0.7	$e > 0$	$e > 0.171$	$e > 0.048$
0.8	$e > 0$	$e > 0.114$	$e > 0.018$
0.9	$e > 0$	$e > 0.057$	$e > 0.004$

1. $A(e)$ is the region of e such that the total cash position is greater in the case of the broad-base contract.
2. $B(e)$ is the region of e such that the futures position is greater in the case of the broad-base contract.
3. $C(e)$ is the region of e such that the expected utility for the hedger is greater in the case of the broad-base contract.