

**INSTITUTE FOR PUBLIC POLICY AND BUSINESS RESEARCH
THE UNIVERSITY OF KANSAS**

**AN ECONOMIC ANALYSIS
OF THE FIRE AT AMERICOLD**

by

Robert H. Glass
Research Economist

A Report to

Kansas State Fire Marshal Department

Mohamed El-Hodiri
Acting Co-Director, IPPBR
Charles Krider
Acting Co-Director, IPPBR

May 1994
Monograph Number 215

AN ECONOMIC ANALYSIS OF THE FIRE AT AMERICOLD

Introduction

During the early morning of December 28, 1991, a fire was reported in the underground storage facilities at the Americold Complex in Wyandotte County Kansas. The fire, the damage created by the fire, and the economic impact of the fire and its aftermath have raised serious questions about underground facilities. The extent of the damages from the fire point to two conflicting aspects of underground facilities. First, these facilities are enormous and provide cost effective space for firms. They are clearly economically significant in the Kansas City Metropolitan Area. Second, a disaster at these facilities can be extensive, and because of their size, the damage can have far reaching effects. This study provides a partial economic analysis of the fire at the Americold Complex. The analysis will concentrate on three economic aspects of the fire: the damage done by the fire at Americold, the government expenses involved in examining the damage created by the fire, and the economic impact of the fire and aftermath on both the Kansas economy and the Wyandotte County economy.

Our analysis is presented in five parts. (1) We present a brief history of the Americold fire. (2) We characterize the difference between the damage done by the fire and the economic impact of the fire. (3) We estimate the cost generated by the fire. Three types of cost are estimated: the damage to inventory kept at Americold, the damage to the Americold warehouse, and the fire equipment lost in the fire. (4) We estimate the expenses incurred by the governmental agencies employed to investigate the fire and its damage. (5) We estimate the direct effect of the fire in terms of the number of jobs lost and the resulting loss of wages and salaries. We also discuss the other direct effects such as the loss of property tax because of the damage to the Americold facilities and the change in Americold's utility billings. (6) We briefly explain the process we use to estimate the indirect effect of the fire and provide estimates of indirect effects of lost jobs and wages and salaries. The indirect effect of changes in property tax and public utilities revenue is not clear and will not be included in this study.

This study is a partial economic analysis because we do not analyze all the economic effects of the fire. We have limited the scope of this study because of time and money constraints. However, to illustrate the broad economic influence of the fire, we mention three economic effects of the fire which will not be analyzed in this study.

The loss of nearly a 150 million pounds of food should have some price effect on food in the United States. If the food were just raw food stuffs, then one could argue that because the

United States has a surplus of food, the effect of losing this food would be insignificant. However, this food had already been processed, at least to some extent, and was packaged. Thus, we expect that this loss of packaged and processed food had some marginal price effect.

The large insurance claims filed because of the fire have for the most part been settled out of court and the results are confidential. Whether or not these settlements were large, from the insurance company's point of view, the probability of large scale damages from an underground fire must have increased. This change in expectations should raise the insurance rates for other underground facilities. The relatively cheap cost of underground storage is one of its major advantages. If insurance rates are raised significantly, then this advantage will be diminished.

The Kansas City Star has reported that millions of pounds of food, which were or should have been embargoed, reached the American public and foreign markets.¹ If the inspectors were correct, and their track record so far is very good, then the consumption of this food will probably create additional medical expenses.

Brief History of the Fire at Americold

The underground area owned by Americold Services Corp. of Portland, Oregon, known as the Americold Complex, has been a warehouse for about 40 years. The warehouse area, approximately 100 acres, was created by limestone mining. The Kansas City area in general, and Wyandotte County in particular, has ledges of limestone located near the surface which make limestone mining feasible. The ledge in Wyandotte County is known as the Bethany Falls ledge, and the underground space was created by mining this ledge and using the limestone in asphalt, concrete, and other materials. The mining began in 1913 with the original mine known as the Peerless Mine. In the 1950's, refrigeration was added to the empty space created by the mine to make the area a commercially viable warehouse facility. In the late 1960's, the area was annexed by Kansas City, Kansas.

The Americold fire began sometime during the night of December 28, 1991. At 2:20 a.m., a burglar alarm in the Return, Inc. storage area was activated. The Kansas City, Kansas police, who had a group of officers searching a wooded area near the Americold facility for the body of a carjacking victim, sent an officer from that group to respond to the alarm. The officer determined that the problem was not a burglar, but a fire, and at 3:07 a.m., the officer requested

¹*The Kansas City Star*, December 12, 1993: pp. 1A, 16-17A; December 13, 1993: pp. 8-9A; January 10, 1994: pp 1A, 10A; January 30, 1994: pp. 1A, 12A.

assistance from the Kansas City, Kansas Fire Department. The difficulties the fire department faced in fighting this fire have been well documented elsewhere and will only be briefly listed: (1) the most direct path to the Americold Complex was blocked by the police search; (2) Americold workers were unable to pinpoint where the fire was; (3) the smoke made navigation and breathing extremely difficult in the caves; (4) firefighters equipment (air tanks, radios, etc.) proved to be inadequate for the underground environment; and (5) the air fan used to ventilate the complex quickly clogged and failed to function. After 14 hours of firefighting, the Kansas City, Kansas Fire Department withdrew from the cave. An indication of the difficulty faced by the fire department was their inability to exactly determine the fire site.

Americold and its insurers then hired a private firm to extinguish the fire. The firm blocked off the area and pumped carbon dioxide into the cave. On January 13, 1992, the blocked off area was checked and the firefighters determined that the fire was still burning. A new injection of carbon dioxide was made and the area was checked again on January 18, 1992. The fire was still burning. The cave was recharged again with carbon dioxide and the firefighters waited. On approximately April 10, 1992, it was determined that the fire was out. By August, the walls of the cave still recorded temperatures around 100 degrees: the normal reading is about 57 degrees. In August, while inspectors were in the cave, a rock, approximately five foot in diameter, fell from the ceiling between two inspectors. After determining the surface temperature of the rock, 130 degrees, the inspectors decided temporarily to end the investigation. The section of the cave where the fire took place has been sealed off, and given the condition of that part of the cave, some inspectors have suggested that it is doubtful if it will ever be know how the fire started.

The Distinction Between Damage Done and Economic Impact

The reader will notice an obvious difference in the order of magnitude between the estimated cost of the fire and the economic impact of the fire. The damage caused by the fire runs into the hundreds of millions of dollars while the fire's impact on Kansas and Wyandotte County is less than ten million dollars. The difference stems from the point of view. Economic impact is concerned with the change in a few selected variables, usually income or employment, within a selected area as the result of some change in the economic environment. Damage is simply the measurement of loss (usually in some currency measure) due to some event.² We will

²The distinction can be explained in terms of stocks and flows. Flow variables are measured over a period of time; such as income per year. Stock variables are measured at a particular

use a simple example to help distinguish between damage and economic impact.

Consider a \$100,000 house that burns down in Kansas City, Kansas. The owner loses \$100,000. Now suppose the owner has insurance and receives \$80,000 from the insurance company for the loss of the house. The owner is out \$20,000 which makes the impact much less than \$100,000. Suppose the owner takes the \$80,000, goes to the bank and borrows another \$20,000 and builds a new house. A Kansas City, Kansas construction firm is hired with only Kansas City, Kansas residents as employees. Let's say that materials represent \$60,000 of the cost and are all purchased from sources outside of Kansas City, Kansas while the rest of the \$100,000 is split between labor and the construction firm as profit. Now the owner is \$20,000 in debt but an additional \$40,000 of income are circulating in the Kansas City, Kansas economy. This is the direct effect of the house fire. However, this is not the end of the economic impact. As this \$40,000 flows through the Kansas City, Kansas economy it creates further income before it leaks from the economy. For example, some of the labor income will be used for things like food. This is usually referred to as the indirect effect. To capture the total impact, an estimate of the indirect effect of the house fire must also be made. This is usually done with a multiplier, and for this example, we will assume the appropriate multiplier for the Kansas City, Kansas economy is 1.5. The direct effect is \$40,000 so the total effect is $(1.5 \times \$40,000)$ \$60,000 which means the indirect effect is \$20,000.

The fire burns down a \$100,000 house, causes \$100,000 in damage and costs the insurance company \$80,000. The owner has to borrow an additional \$20,000 to construct a new house which puts \$40,000 into the Kansas City, Kansas economy which creates an additional \$20,000 in income. Net results: damage is \$100,000, increased debt is \$20,000, and economic impact is \$60,000 in increased income. Which number is the proper measure of the economic effect of the house fire? Again it depends on one's point of view. For the house's owner, the insurance company and the fire department, the damage figure best describes the economic effect on them. The owner and the insurance company are out money based on the damages, and the fire department is burdened with the responsibility of taking appropriate action to protect \$100,000 worth of property. For the tax collector, the best measure is the increase in taxable income. For the community as a whole, we feel that both the damage done and the economic impact should be considered in order to give a more complete picture of the economic effect of the fire.

point in time; such as wealth. Economic impact is the change in the flow of income or the change in the demand for labor during a year (employment). Damage is the loss in the inventory stock or the change in the stock of wealth.

Damage

The fire created extensive damage. We will only estimate damage figures for three areas: the damage to the Americold warehouse, the damage to the inventory at Americold, and the loss of equipment by the Kansas City, Kansas Fire Department.

Americold Warehouse

The extent of the damage at the Americold Complex can only be inferred. Because of the fire, Portal A was closed. Portal A is about 14 acres while the whole warehouse is about 100 acres. The Wyandotte County Appraiser's Office believed that prior to 1992, the Americold Complex was severely undervalued for property tax purposes. As a result, they reappraised the Americold Complex in late 1991. The preliminary 1992 appraised value for the 21 parcels of property owned by Americold was \$15,193,180, more than double the 1991 appraised value. Even before the fire, Americold challenged this appraised value. After the fire, Americold got the appraised value reduced to \$13,727,700 in 1992 of which \$12,169,440 represented the value of the property where the warehouse is located. We have not been able to determine how much of the reduction in appraised value was due to the fire; however, \$1,000,000 is probably conservative and not too far off.

Inventory at Americold

Different estimates have circulated of the damage inflicted by the fire on the inventory at Americold. According to the *Kansas City Star*, on December 28, 1991, the Americold Complex contained 276.5 million pounds of food from more than 100 companies — this was enough food to fill about 72.9 miles of tractor-trailers. The food stored at Americold included more than 52 million pounds of meat and poultry, one million pounds of walnuts, and 60 million pounds of tomato paste among other items.³ Due to contamination, 66.7 million pounds of United States Department of Agriculture food had to be destroyed. Of the remaining approximately 190 million pounds of food, 79.6 million pounds also had to be destroyed with about 10 million pounds of food waiting disposition. This was in the December 12, 1993 *Kansas City Star*.

These figures are probably close, but they are not the final estimate according to Roger

³The *Kansas City Star*, December 12, 1993: p. 16.

Ozias, the Kansas Department of Health and Environment inspector who has headed most of the inspection. He notes there may be some double counting with the U.S. Department of Agriculture. He will not make an official estimate until some disposition has been made on all the remaining food — approximately 4 million pounds of food is waiting disposition. He expects to have a final estimate in a couple of months.⁴ Even with this uncertainty, it seems clear that a sizable portion of food was destroyed, and the figure of 150 million pounds is probably going to be in the ballpark. Given the variety of the food items, guessing as to the value of this food seems hazardous.

Kansas City, Kansas Fire Department

Table 1 contains a listing of the equipment lost by the Kansas City, Kansas Fire Department at the Americold fire.

Expenses Involved in Government Investigation

Because of the size and scope of the Americold fire, numerous governmental agencies at the federal, state, and local level were involved in investigating the fire and its damage. However, we will only provide estimated expenses for five different governmental agencies. At the state and local level we will estimate the cost to the Kansas Department of Health and Environment and the Wyandotte County Health Department. At the national level we will estimate the cost to the Environmental Protection Agency, the Food and Drug Administration, and the Occupational Safety and Health Administration. Table 2 below contains the conservative estimates given to us by each of these agencies. The total cost to governmental agencies was approximately \$200,000. This ignores one of the major contribution to analyzing the fire: the work of the Kansas City, Kansas Fire Department.

⁴Phone conversation with Roger Ozias, March 23, 1994.

Table 1

**Fire Department
Equipment Lost at Americold**

Equipment	Dollar Value		
Rope Bags	\$49.00	Rope	\$360.00
Honda Portable Generator	\$970.00	Salvage Cover	\$99.95
Air Bottles	\$5,665.00	Hand Lights	\$71.80
1 3/4 Inch Nozzles	\$2,869.65	Positive Pressure Fans	\$2,996.85
2 1/2 to 1 1/2 Gateways	\$356.00	Complete Air Packs	\$5,994.00
Plug Wrench	\$18.85	Air Masks	\$894.50
Pry Bar	\$29.29	Spanners	\$15.90
4 Foot Pike Pole	\$28.00	Extension Cords	\$60.00
Gas Cans	\$12.00	Scoop Shovels	\$59.00
2 1/2 Fog Nozzle	\$624.95	2 1/2 Straight Bore	\$624.95
Spanner Straps	\$281.70	1 3/4 Inch Hose	\$4,425.00
2 1/2 Inch Hose	\$5,725.00	3 Inch Hose	\$8,025.00
Total	\$40,256.39		

Table 2

State and Local Governmental Agencies	Expense
Wyandotte County Health Department	\$10,000
Kansas Department of Health and Environment	\$125,000
Federal Agencies	
Environmental Protection Agency	\$20,000
Food and Drug Administration	\$27,000
Occupational Safety and Health Administration	\$17,000
Total	\$199,000

The Direct Effects

The Americold facility includes a limestone quarry, a records center, and a warehouse. Operations at the limestone quarry appear unaffected by the fire. The records center was affected, but only employed 8 people. For this analysis, we will ignore the records center.⁵ The warehouse is a storage and distribution center divided into public and leased space. The public space is maintained by Americold and, at the time of the fire, Americold employed 140 people there. The leased space is maintained by the companies which lease the space and, at the time of the fire, they employed 541 people. Lew Levin's report, "Kansas City, Kansas Americold Facility, Economic Impact Analysis," referred to hereafter as "Americold Analysis", provided this information, was invaluable for our research, and was our starting point.

"Americold Analysis" states that Americold laid off 88 employees as a result of the fire and recalled 33 in April primarily for clean up. *The Kansas City Star* on June 3, 1993 reported that the work force was down to about 70 employees. Based on this evidence we will assume that half the warehouse labor force at Americold in 1991 (70 employees) lost jobs because of the fire. "Americold Analysis" identifies five major leasing firms which employed 362 of the 541 people employed in the leased area of the warehouse. (See Table 3.) Of these firms, we know that Kraft Foods and Safeway stores transferred their operations outside of Wyandotte County and Kansas. These two firms employed 97 people and their movement represents a loss to both the county and the state. Two firms moved from the Americold facility but stayed in Wyandotte County or are making arrangements to keep their operations in Wyandotte County which has no impact on either the county or the state. Finally, one firm remains at Americold. Little is known of the smaller firms in the leased space which employed 179 people. One firm which closed was reported to have employed 25 people, but through other sources we have been told that these 25 people really represented only about 15 full-time jobs.

The direct effect of the fire on employment at Americold appears to be a loss of 70 Americold employees and 112 employees in the leased area. Because of the location of Wyandotte County and the inter-county and inter-state commuting patterns of the metropolitan area, one would not expect all of these lost jobs to be held only Wyandotte County residents or even by Kansas residents. "Americold Analysis" reveals that in 1991, 39.9% of the warehouse employees who worked for Americold lived in Wyandotte County, 39.3% lived in Kansas but

⁵Americold officials and their lawyers were asked if any positions from the records center were eliminated because of the fire, but for legal reasons they chose not to answer.

not Wyandotte County, and 20.8% lived in Missouri. (See Table 4.) Even though this data is only for Americold warehouse employees, we will use these ratios to allocate all of the lost jobs among the three categories of residents. As a result of the allocation process we estimate that of the 182 people who lost their jobs, 73 were residents of Wyandotte County, another 72 were Kansas residents but not Wyandotte County residents, and 37 were Missouri residents.

Again because of the work of Mr. Levin, we are able to estimate the impact of these lost jobs on wages and salaries. The 1991 estimated average income of Americold's warehouse employees was \$28,789.29. The 1991 estimated average income of the employees who worked for firms which leased space was \$25,000. In addition, the fringe benefits of these employees was estimated at 25% of their income. We used the same ratios to allocate the residence of lost wages and salaries as we used above to allocate the residence of lost jobs. The result of the allocation process was an estimate that Wyandotte County residences lost \$2.40 million in wages and salaries, Kansas residents who were not Wyandotte County residents lost \$2.37 million in wages and salaries, and Missouri residents lost \$1.25 million in wages and salaries.

Table 3

Employment at the Americold Complex: 1991

Public Space	148
Leased Space	541
Swift-Eckrich	100
Fleming Foods	125
Wilson Foods	40
Kraft Foods	82
Safeway Stores	15
Other Leased Space	179

Table 4

**Employment in the Americold Warehouse
by Residence**

Residence	Payroll	% of Total	Employment
Wyandotte County	\$1,701,004	42.2%	71
Other Kansas Residence	\$1,613,200	40.0%	70
Missouri Residence	\$715,727	17.8%	37
Total	\$4,029,931		178

We mention two second order direct effects of the fire: the loss in property tax and the effect on utility billing. The Wyandotte County Appraiser's Office believes that prior to 1992, the Americold facility was severely undervalued for property tax purposes. The preliminary appraised value for 1992 for the 21 parcels of property owned by Americold was \$15,193,180, more than double the 1991 appraised value. Even before the fire, Americold challenged this appraised value. After the fire, Americold got the appraised value reduced to \$13,727,700 in 1992 of which \$12,169,440 represented the value of the property where the warehouse is located. We have not been able to determine how much of the reduction in appraised value was due to the fire; however, \$1,000,000 is probably not too far off. This represents a loss of about \$45,000 in property tax. Utility billings also changed significantly from 1991 to 1992 for the Americold facility. Water billings increased about \$50,000, more than doubling, while electric billings dropped about \$150,000.

The Indirect Effect

We used the KSSAM2 (Kansas Social Accounting Matrix) model of the Kansas economy developed by the Institute for Public Policy and Business Research to estimate the indirect effect of the fire on the Kansas economy. A social accounting framework such as KSSAM2 is an extension of an input-output model for a region. The major difference between KSSAM2 and an input-output model is that household and government behavior are exogenous in an input-

output model while within the social accounting framework, they are endogenous (i.e. taken into account). For the purposes of estimating an economic impact, the KSSAM2 model is solved for a 52 by 52 matrix of multipliers. These multipliers show the total effect in each of 52 sectors, from a given direct effect in each sector. Therefore matrix multiplication on the vector of direct effects leads to a vector of total effects. Each element in the vector of total effects represents the total effect of the fire on each of the 52 sectors in the model.

The indirect effect of the fire on the Wyandotte County economy was estimated by developing a model similar to the KSSAM2 using Wyandotte County data. The only structural difference in these two models is that the Wyandotte County model only has one endogenous governmental sector (local government) while the KSSAM2 model has three governmental sectors (state excluding education, public education, local excluding education). Thus, the Wyandotte County matrix of multipliers is only a 50 by 50 matrix. However, the same procedure is used to calculate the total effect of a direct effect in the Wyandotte County model as in the KSSAM2 model.

Results of the Estimation Process

The aggregated results of our estimation of the direct effect, indirect effect, and total effect of the Americold fire in terms of lost income and jobs are presented in the Table 5. These figures were calculated using a complete matrix of multipliers. The income to income multipliers are 1.88 for the Kansas economy and 1.44 for the Wyandotte County economy. The income to income multiplier means that a one dollar exogenous decrease in income to households results in an additional indirect decrease of Kansas personal income of 88 cents. For Wyandotte County personal income, the one dollar decrease in income means an additional indirect decrease of Wyandotte County income of 44 cents. Thus, combining the direct effect and the indirect effect gives a total effect of a decrease of \$1.88 in Kansas personal income due to a decrease of one dollar in Kansas personal income, and a total effect of \$1.44 decrease in Wyandotte County personal income due to a decrease of one dollar in Wyandotte County personal income. The reason the Kansas economy's multiplier is larger than the Wyandotte County economy's multiplier is because the Kansas economy encompasses the Wyandotte County economy while being significantly larger; thus, it takes longer for initial impacts to leak out of the Kansas economy than the Wyandotte County economy. The multiplier captures the effect of dollars spent in an economy before they leave the economy. The larger the economy, the longer the dollars will stay in that economy.

The job to job multiplier is similar to the income to income multiplier. A decrease of one Kansas job at the Americold facility causes the loss of an additional 0.75 job in the rest of the Kansas economy, and a decrease of one Wyandotte County job causes the loss of an additional 0.68 job in the rest of the Wyandotte County economy.

Table 5

**The Economic Impact of the Fire at Americold
(In 1991 Dollars)**

The Impact on the Kansas Economy

	Personal Income	Jobs
Direct Effect	\$4,770,000	145
Indirect Effect	\$4,190,000	126
Total Effect	\$8,960,000	271

The Impact on the Wyandotte County Economy

	Personal Income	Jobs
Direct Effect	\$2,400,000	73
Indirect Effect	\$1,060,000	56
Total Effect	\$3,460,000	129

APPENDIX

Introduction

This appendix provides both a formal mathematical description and a verbal description of the mathematical model used to study the economic impact of warehouse fire at Americold on the Wyandotte County and Kansas economy. Specifically, this appendix will discuss, in a non-mathematical manner: input-output modeling; the construction of a social accounting model like those used in this study; and the empirical results of the investigations along with a more technical description of input-output modeling and the social accounting model used in this study.

The Total Effect of an Economic Change

Direct and Indirect Effects of Economic Change

When a change occurs within an economy it causes other modifications within that economy. These secondary or indirect modifications of the economy are the result of the structural interdependence of most sectors of a modern economy. These two effects are generally referred to as the direct and indirect effects of an economic change. The initial change referred to as the direct effect on the economy and is usually easily observed. For example, the laying off of 100 employees with combined annual wage and salary income of \$4,000,000 would be a direct effect. As this direct effect reverberates through the economy, it causes further change in that economy: indirect effects. For example, the loss of \$4,000,000 in personal income in an economy will result in lower expenditures for retail sales and, if a sales tax exists, less revenue from that tax. Other indirect effects would be a decline in demand in the automobile and housing markets and reduced revenue from an income tax. These indirect effects are not immediately or directly observable, but they penetrate into every sector of an economy.

Methods for Generating Multipliers

The magnitude of these indirect effects can be estimated through the use of multipliers. A multiplier gives a quantitative estimate of the indirect effects of an initial change in an economy, based on the source and the size of the initial change. Regional multipliers can be estimated using three basic methods. The first is the economic base method, which in its simplest form is similar to a simple open Keynesian model for a region. Unfortunately, this type of model operates at an aggregate level and does not provide the ability to analyze the effects of changes

at the industrial level. An econometric model is the second possible source for developing regional multipliers. The most detailed econometric model of the Kansas economy has been developed by IPPBR (its name is KEM). Despite this model's complexity — it forecasts over 100 Kansas variables — it does not have the capability to generate the appropriate multipliers. The third method of estimating multipliers, and the method we have chosen to use, involves a modified input-output model. This type of model provides detailed, inter-industry multipliers. Numerous papers have also documented that when the detailed estimates of an input-output model are aggregated, the aggregate multipliers are similar to the economic base multipliers [Pleeter 1980, p. 27].

Simple Input-Output Models

Explanation of the Simple Input-output Model

The simplest input-output model is a series of industrial supply and demand relations treating industries and consumers as the only economic agents. Supply is the output of each industry. Demand for this output is separated into two major components: final demand and intermediate demand. Final demand is the demand for goods and services by individual consumers for their own use. Intermediate demand is the demand for goods and services by one industry for use in the production of other goods and services. For example, the use of steel by an aircraft firm to produce airplanes is an intermediate demand for steel. By the same token, the use of a company airplane by the sales staff of a steel firm for the generation of steel orders is another example of intermediate demand.

The assumption that all goods and services produced in this economy must be used creates the simple accounting identity: an industry's output is equal to the uses of that output as an input to production plus the consumption of that output by individuals. In simpler economic terms, supply is equal to intermediate demand plus final demand for each industry. Thus, the identity yields a series of industrial supply and demand equations. In input-output models we stack these industrial supply and demand relationship on top of each other, number each industry, and use this numbering system to sequence, in each supply and demand equation, the individual intermediate demands for output. For example, suppose the electrical utility industry's supply and demand equation is fifth from the top in the stack of supply and demand equations. Then in each supply and demand equation, the electrical utility's intermediate demand is the fifth intermediate demand from the beginning of the intermediate demands.

By maintaining the industrial order, the stack of industrial supply and demand equations can compactly be depicted by a matrix equation. In terms of matrix algebra, a vector of outputs (the

stack of industrial outputs) is equal to a square matrix of intermediate demands (the stack of intermediate demands) plus a vector of final demands (the stack of final demands). This matrix relationship is the heart of any input-output model. The column vectors both have the same height (the same number of entries) as the number of industries within the model. The square matrix's dimensions are also determined by the number of industries within the model. If the model has ten industries, then the column vectors are ten entries high and the square matrix has ten columns and ten rows.

The matrix of intermediate demands makes the input-output model unique (and gives it its name). This matrix is a matrix of inputs to production which captures the inter-industry flow of goods and services necessary for production: a matrix of inputs to produce a vector of outputs. A column in this matrix represents the various inputs to production used by one particular industry. In the example of the electrical industry used above, the fifth column represents the inputs necessary for the production of electricity. These inputs are generated by the production of the other industries in the economy. A row of this matrix represents the uses of one industry's production in the production processes of all the industries in the economy. Again using the example above, the fifth row represents the use of electricity generated by the electric utility industry in the production of all other industries. Obviously, in a real economy, some of the cells in the matrix will be zero; that is, some industries will not use the production of other industries in their own production process.

Making the Model Operational

This description of the simple input-output model captures the basic concept of the model. However, to make the model operational, a couple of alterations are necessary. The variables in the model above were described in physical terms, which implies measurement in physical units such as number of hamburgers and tons of steel. In order for the input-output model to be flexible, the output, inputs and final demand for all industries need to be measured in the same units. This is accomplished by substituting the monetary value of the goods and services produced and used for their physical units. Instead of tons of steel produced, the model now works with the dollar value of all the steel produced, and instead of the number of hamburgers eaten, the model works with the dollar value of hamburgers eaten.

A second alteration in the model is necessary to generate multipliers. Each column in the matrix of inputs to production represents the monetary value of inputs to the production of an industry. If each of the inputs to the production of a particular industry (each column entry) is divided by that industry's dollar value of output, a column of input coefficients is created. If this process is followed for all industries, then a matrix of production coefficients (or input

coefficients) is created. The assumption made by economists who work with input-output models is that these production coefficients are an adequate description of the production process, except for labor, at all relevant levels of production. At this point of development, the contribution of labor to the production process is being ignored. These production coefficients are not going to adequately describe the production process at all levels of production, but if the band of expected change is relatively small (less than a 25 percent change in output), then these coefficients are an adequate representation of the production process [Miller and Blair 1985, pp. 266-316].

A second important assumption implied by the use of the matrix of production coefficients is that price ratios are constant. This assumption is necessary because these coefficients are derived from the value, in dollars, of inputs to production. If price ratios change, then the production coefficients would change even if the production process still required the same physical amounts of all goods and services. In addition, if relative prices change, then probably some substitution among the goods and services used as inputs would take place. All of the above means that if the assumption of constant price ratios is violated, then the production coefficient matrix may need to be adjusted. (In a special case referred to as "Cobb-Douglas Technology", changes in quantities just offset any changes in prices so that dollar values remain unchanged.)

The construction of the matrix of production coefficients by dividing all the industrial columns by their industrial output creates an imbalance in the basic supply and demand relationships. To rectify this imbalance, the coefficient matrix must be multiplied by the column vector of outputs. An analogy with a single equation supply and demand model should help explain why this manipulation is necessary. Suppose one has an equation such that output is equal to intermediate demand plus final demand. If intermediate demand is divided by output, then in order to maintain the equality, intermediate demand must also be multiplied by output. This is the old algebra trick of multiplying by one; that is, multiplying by output divided by output.

Derivation of Multipliers

The substituting of the intermediate demand matrix (the matrix of inputs to production) with the production coefficient matrix multiplied by the vector of industrial outputs creates in the simple input-output model the opportunity to generate a matrix of multipliers by algebraically solving the model. Appendix 3-2 provides the procedure for solving the model and generating the matrix of multipliers. The algebraic result of solving the model is that the column vector of industrial outputs is equal to a matrix of multipliers multiplied by a column vector of final demands. (Matrix multiplication must be used in the above relationship.) The matrix of

multipliers is the result of subtracting the matrix of production coefficients from the identity matrix and then inverting the result.)

The matrix equation, in which the column of outputs is equal to the matrix of multipliers multiplied by the column of final demands, is the basis for our benefit analysis. This equation links final demand to output. If this relationship holds true at levels of economic activity near the equilibrium level of output and final demand, then changes in final demand are linked to changes in output. The simple mathematical equation does not establish a cause and effect relationship between final demand and output, it only establishes that these variables are related. The cause and effect relationship can only be established in the construction of the mathematical model. Economists distinguish between variables that are exogenous to a model and variables that are endogenous to a model. The values of the exogenous variables are determined outside of the model. The values of the endogenous variables are determined within the model. The relationship between the exogenous and endogenous variables in an economic model reveals the assumed cause and effect relationship within the model.

In the case of this simple input-output model, the equation described above is interpreted as operationally meaning that a change in the column vector of final demands multiplied by the matrix of multipliers yields the change in industrial output. Because the multipliers are derived from the matrix of production coefficients, which in turn were derived from the matrix of inter-industry flows of goods and services, these multipliers take the initial, direct effect on final demand and create an estimate of the direct plus indirect effects on industrial output due to an exogenous change in the economy.

Enhanced Input-Output Models

Incorporating Investment

The simple model is missing many obvious elements of a real economy. In the next two sections, we will add to this simple model to endow it with more realism. The first major addition will be an investment sector. In a real economy, not all of the goods used by firms in production are materials that go into the product. Some of the goods are used as investment. The process used to create the matrix of intermediate demand can be used to create a matrix of investment demand. Essentially demand is split between intermediate demand (operational demand) and investment demand. The matrix of investment demand has the same dimensions as the matrix of intermediate demand. By using market prices, investment demand can also be put into monetary units. By dividing the investment demand matrix by sector output, an investment coefficient matrix is created just as in the case of intermediate demand. Multipliers

are calculated in a similar fashion to the simple model; however, the resulting matrix that must be inverted has both the investment coefficient matrix and the intermediate demand coefficient matrix subtracted from the identity matrix. Incorporating investment in the simple input-output model simply means adding another matrix to the basic matrix equation.

Social Accounting Framework

The social accounting framework endogenizes more of the variables in the simple input-output model. The simple input-output model with investment consists solely of output, a production process, and investment process, and exogenous final demand. The social accounting framework endogenizes households and government. The model will still have the same basic structure as the simple model with investment added— consumption plus investment plus exogenous final demand is equal to generalized gross state product; however, the individual components will be more complex. The social accounting framework also allows for an analysis of the distributional effects of exogenous changes. In an input-output model, households are one sector; however, in a social accounting framework, households can be separated into different classes with different value added coefficients, consumption coefficients, and investment coefficients. Government can also be unbundled into different jurisdictions.

Endogenizing households

To endogenize households in the model, income to households, the consumption of goods and services by households, and investment by households need to become part of the circular flow within the economy—these factors need to be part of the feedback process in the model. The production coefficient matrix created above does not include labor costs or other value-added costs of production. To add labor cost to the model, consider the dollar value of all labor needed in each sector. Each sector's labor requirement can be converted into a coefficient as the other production inputs were before by dividing the dollar value of labor needed for production by the value of output in each respective industry. Multiplying these coefficients by any change in output produces the resulting change in labor income due to the change in output. This increase in labor income is an increase in income to households.

How households spend this income depends upon the pattern of household expenditures on goods and services. The vector of consumer demand for goods and services provides this pattern. By dividing each element in this vector by personal income, a vector of coefficients is constructed which represents the share of personal income spent on each sector's output. This vector of coefficients is a vector of consumption shares. The same can be done with household

investment to create a matrix of household investment coefficients like the business investment coefficients created in the last sector. Tying together labor income, household consumption, and household investment adds new feedback facets to the model. The change in output causes a change in labor income which then causes a change in household consumption and investment.

Endogenizing government

For government to be part of the feedback loop in the model, government expenditures and taxes, and government investment all need to be part of the circular flow of the model. The same procedure is followed with government as with households to endogenize them. Rather than being lump sums, taxes and expenditures become coefficients multiplied by output. Government investment is handled as investment is handled for all other sectors.

The effect on the multipliers

A qualitative result of endogenizing households and government is to alter the multipliers. The multipliers derived before only took account of the indirect effect on output as the direct effect moves through the production process. The new multipliers include the influence of the indirect effect on household consumption and investment through changes in wages and salaries which affect household income; in other words, these new multipliers capture the feedback effect on households of a change in exogenous final demand. These new multipliers also capture the influence of changes in output on government income through taxation and the effect on expenditures of changes in income. These effects are called the induced effects because they result from the additional income for households and government induced by the direct effect. The multipliers which are derived from a model where households and government are "endogenized" capture three effects: direct, indirect, and induced. Endogenizing households makes the multipliers larger [Miller and Blair 1985, pp. 100-105].

A detailed mathematical description of our model, its solution, and the generation of its multipliers is provided in Appendix B. The matrix techniques necessary for the solution of our model and the generation of the multipliers are similar to the techniques used to solve the simple input-output model and generate its multipliers. The result is the same: a matrix of multipliers which links a change in final demand to a change in industrial output.

Empirical Aspects of Multipliers

Data Sources

The data used to generate the parameter values for our model came from the U.S. Input-Output Tables for 1977 and 1985 developed by the U.S. Department of Commerce, Bureau of Economic Analysis (BEA); personal income, farm income, and crop and livestock cash receipts for the state and various counties developed by BEA; and employment and payroll estimates produced by the U.S. Department of Commerce, Bureau of the Census, *County Business Patterns*.

The Structure of the Social Accounting Model

The social accounting models used in this study is based on the KSSAM2 model: a social accounting matrix for Kansas. The model has 55 sectors: 48 business sectors, 4 household sectors, and 3 government sectors. However the household sectors were collapsed into one sector because distribution issues were not important for this study and in the case of Wyandotte County, the government sectors were collapsed into one sector. A list of the sectors along with their definitions is given in Table 6.

Table 6
Sector Definitions of the Social Accounting Model

<i>#</i>	<i>Sector Definition</i>	<i>Included SIC Codes</i>
1	Livestock	021, 024, 025 except 0254; 027, 029
2	Crops	01
3	Forestry, Commercial Fisheries	08
4	Agricultural Services	0254, 07 except 074; 085, 09
5	Metal and Nonferrous Mineral Mining	10
6	Coal Mining	1111, 1211
7	Oil and Gas Extraction	131,132
8	Stone, Clay, and Gravel	141, 142, 144, 145, 149
9	Construction	15-17 except 153; 1112, 1213, 138, 148
10	Food Processing	20
11	Tobacco Processing	21
12	Fabrics and Apparel	22, 23
13	Lumber and Wood	24 except 2451
14	Furniture and Fixtures	25
15	Paper Products	26

Table 6 (Continued)

<i>#</i>	<i>Sector Definition</i>	<i>Included SIC Codes</i>
16	Printing and Publishing	27
17	Chemicals	147, 281, 286, 287, 289
18	Plastic Materials and Synthetics	282
19	Drugs and Preparations	283, 284
20	Paints	285
21	Petroleum Refining	29
22	Rubber, Rubber Prod., Plastic Prod.	30
23	Leather and Leather Products	31
24	Glass, Stone, and Clay Products	32
25	Iron, Steel, and Other Metal Prod.	33, 3463
26	Metal Prod., Ordnance, Struct. Met	34 except 3463; 3761, 3795
27	Engines and Machinery	35 except 357
28	Computers, Computing Equipment	357
29	Electrical Equipment and Appliances	36 except 367; 3825
30	Electronic Components and Parts	367
31	Motor Vehicles and Equipment	371 except 3716
32	Aircraft and Parts	372, 376 except 3761
33	Other Transportation Equipment	2451, 3716, 373, 374, 375, 379 exc. 3795
34	Scientific and Photographic Equip.	38 except 3825
35	Misc. Manufacturing	39
36	Transportation and Warehousing	40-42, 44-47
37	Communications Except Radio and T.V.	48 except 483
38	Business Services, Radio and T.V.	483, 73 except 7396; 8100, 89 exc. 8920
39	Electric Services, Utilities	49 except 491
40	Wholesale and Retail Trade	50-57, 59, 7396, 8042
41	Finance and Insurance	60-64 except 613; 67 except 6732
42	Real Estate and Rental	153, 65, 66
43	Hotels, Personal Services	70, 72, 76
44	Eating and Drinking Places	58
45	Automobile Repair and Services	75
46	Amusements	78, 79
47	Private Health, Ed., and Soc. Serv.	074, 6732, 80 exc. 8042; 82, 84, 86, 892
48	Federal Government Enterprises	43, 613
49	Household	N/A
50	State government except education	N/A
51	Local government except education	N/A
52	Public education	N/A

Changes in Income and Employment

Table 5 has the aggregated effect of the Americold fire on income and employment.

The Formal Model

A Generic Input-Output Model

The construction of an input-output model begins with a series of individual industry supply and demand equations. For example, consider industry i ,

$$(1) \quad \phi_i = \xi_{i1} + \xi_{i2} + \dots + \xi_{ij} + \dots + \xi_{in} + \gamma_i$$

where ϕ_i is the output for industry i in physical units
 ξ_{ij} is the intermediate quantity demand for industry i 's output by industry j
 γ_i is the final demand for industry i 's output

These equations assume that all output is used as an input to further production or is consumed as final demand. The quantities in these supply and demand equations are measured in physical terms rather than by monetary value. For example, the number of planes used or the tons of steel produced.

Stacking all the industrial supply and demand equations in order provides the following configuration of equations:

$$(2) \quad \begin{aligned} \phi_1 &= \xi_{11} + \dots + \xi_{1j} + \dots + \xi_{1n} + \gamma_1 \\ &\vdots \\ \phi_i &= \xi_{i1} + \dots + \xi_{ij} + \dots + \xi_{in} + \gamma_i \\ &\vdots \\ \phi_n &= \xi_{n1} + \dots + \xi_{nj} + \dots + \xi_{nn} + \gamma_n \end{aligned}$$

These industrial supply and demand equations can be converted to a single matrix equation which describes this simple economy:

$$(3) \quad \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_i \\ \vdots \\ \phi_n \end{bmatrix} = \begin{bmatrix} \xi_{11} & \dots & \xi_{1j} & \dots & \xi_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{i1} & \dots & \xi_{ij} & \dots & \xi_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{n1} & \dots & \xi_{nj} & \dots & \xi_{nn} \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_i \\ \vdots \\ \gamma_n \end{bmatrix}$$

A more compact method of writing these relationships is given in Equation (4), which describes the production relationships in this simple economy in terms of physical units: numbers of planes, tons of steel, etc.

$$\Phi = \Xi + \Gamma$$

(4) where Φ is the vector of industrial output
 Ξ is the matrix of intermediate demands
 Γ is the vector of final demand

To proceed with economic analysis, these production relationships need to be expressed in monetary terms.

Changing from measurements in physical units to units of monetary value requires the addition of prices to the model. Let p_i be the market price of the output of the i th industry. Then equation (1) becomes:

$$(5) \quad p_i \cdot \phi_i = p_i \cdot \xi_{i1} + p_i \cdot \xi_{i2} + \dots + p_i \cdot \xi_{in} + p_i \cdot \gamma_i$$

This equation establishes the relationship that the monetary value of output is equal to the intermediate demand for the output plus the final demand for the output, with all variables measured in monetary units. The following new variables make the notation for the model easier to follow:

$$p_i \cdot \phi_i = x_i, \quad p_i \cdot \xi_{ij} = z_{ij}, \quad \text{and} \quad p_i \cdot \gamma_i = y_i$$

Using these new variables, equation (5) now becomes:

$$(6) \quad x_i = z_{i1} + z_{i2} + \dots + z_{in} + y_i$$

where x_i is the output for industry i
 z_{ij} is the intermediate demand for industry i 's output by industry j
 y_i is the final demand for industry i 's output

Stacking all industrial equations in order, as was done with equation set (2), and converting the result to matrix form, as was done with equation (3) gives:

$$(7) \quad \begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} z_{11} & \dots & z_{1j} & \dots & z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ z_{i1} & \dots & z_{ij} & \dots & z_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ z_{n1} & \dots & z_{nj} & \dots & z_{nn} \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix}$$

which can be written more compactly as:

$$(8) \quad X = Z + Y$$

The rows of the Z matrix in equation (8) represent input demand for industry i 's output while the columns of the matrix represent industry i 's input demand for all other output.

Unlike the Z matrix from Equation (4), the Z matrix is a description of the production process of the economy in monetary units. If prices in the economy change, then the Z matrix will change even if the underlying physical production process is unchanged. Put another way, if relative prices change, but the productive structure of the economy remains the same, then the Z matrix will not change, but the Z matrix will change. The Z matrix can be converted into a matrix of coefficients which describe the economy's production process. The production coefficients are created by dividing each column entry by the output of that industry. For example, consider the use of industry i 's output as an input by industry j :

$$(9) \quad \frac{p_i \cdot z_{ij}}{p_j \cdot x_j} = \frac{z_{ij}}{x_j} = a_{ij}$$

This procedure can be followed for all the entries in Z matrix in equation (7) creating a matrix of production coefficients which is commonly referred to as the A matrix.

$$(10) \quad \begin{bmatrix} \frac{z_{11}}{x_1} & \dots & \frac{z_{1n}}{x_n} \\ \vdots & \vdots & \vdots \\ \frac{z_{n1}}{x_1} & \dots & \frac{z_{nn}}{x_n} \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} = A$$

The Z matrix in equations (7) and (8) can not be replaced by the A matrix just created above without some adjustments. Because the A matrix was created by dividing each column by that industry's output, each entry needs to also be multiplied by that industry's output to maintain

the equality in equations (7) and (8). This is the same as post-multiplying by the column vector of outputs.

$$(11) \quad \begin{bmatrix} \frac{z_{11}}{x_1}x_1 & \dots & \frac{z_{1n}}{x_n}x_n \\ \vdots & \ddots & \vdots \\ \frac{z_{n1}}{x_1}x_1 & \dots & \frac{z_{nn}}{x_n}x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 & \dots & a_{1n}x_n \\ \vdots & \ddots & \vdots \\ a_{n1}x_1 & \dots & a_{nn}x_n \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

The right hand side of equation (12) can now be substituted for the Z matrix in equation (7).

$$(12) \quad \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

Equation (12) can be written more compactly by substituting the A matrix post-multiplied by the column vector of outputs into equation (8).

$$(13) \quad X = A \cdot X + Y$$

Solving the above equation for the column vector of outputs generates the multipliers for the model.

$$\begin{aligned} X - A \cdot X &= Y \\ (I - A)X &= Y \\ X &= (I - A)^{-1} Y \end{aligned}$$

The usefulness of the multipliers depends upon whether around this solution (or equilibrium point), a change in final demand will alter output by the same proportion that is established at the equilibrium point. Given this assumption, the solution can be interpreted as:

$$(14) \quad \Delta X = (I - A)^{-1} \Delta Y$$

where ΔX is the change in output
 ΔY is the change in final demand
 $(I - A)^{-1}$ is the matrix of multipliers for a change in final demand

Adding Investment

The simple model has many obvious elements of a real economy missing. In the next few sections, we will add to this simple model to endow it with more realism. The first major addition will be an investment sector. In a real economy, not all of the goods used by firms in production are materials that go into the product. Some of the goods are used as investment. The logic used to create the A matrix can be used to create an investment coefficient matrix- a B matrix.

With the simple model, the underlying physical use of goods was described by Equation (4):

$$\Phi = \Xi + \Gamma$$

Now we add a matrix of investment demands, Π , which has the same dimensions as Ξ . Equation (4) becomes:

$$(15) \quad \Phi = \Xi + \Pi + \Gamma$$

Let the elements of Π be π_{ij} , then by using market prices as in equation (5), a new set of variables is created — $p_i \cdot \pi_{ij} = s_{ij}$, where s_{ij} is the monetary value of investment demand for industry i 's output by industry j . Let S be the matrix of these elements of investment demand, then equation (8) becomes:

$$(16) \quad X = Z + S + Y$$

Just as a production coefficient matrix was created by dividing each z_{ij} by x_j , an investment coefficient matrix can be created by dividing each s_{ij} by x_j . Let B be the resulting investment coefficient matrix with the individual elements being $b_{ij} = s_{ij}/x_j$. Equation (13) now becomes:

$$(17) \quad X = A \cdot X + B \cdot X + Y$$

which can be solved for the matrix of multipliers:

$$(18) \quad \Delta X = (I - A - B)^{-1} \Delta Y$$

where $(I - A - B)^{-1}$ is the matrix of multipliers when investment is added to the simple input-output model.

Social Accounting Framework

The simple model with the addition of investment does not explicitly have any income creation, personal consumption, or government activity. The model consists solely of output, a production process, and investment process, and exogenous final demand. In this section, we will extract households and government from exogenous final demand and make these two sectors endogenous. The increase in reality comes at a cost — the notation becomes more cumbersome and we will have matrices of matrices. The model will still have the same basic structure — consumption plus investment plus exogenous final demand is equal to generalized gross state product; however, the individual components will be more complex.

Consumption is now the consumption of businesses, households, and governments. Businesses are divided into sectors, households are divided into income classes, and governments are divided by jurisdiction. Thus, the consumption matrix, U_1 , is a three by three matrix consisting of nine matrixes.

$$U_1 = \begin{bmatrix} \text{BusCon} & \text{HHCon} & \text{GovCon} \\ \text{BusValue} & \text{HHValue} & \text{PayTran} \\ \text{BusTax} & \text{HHTax} & \text{InterGov} \end{bmatrix}$$

To make this as intuitive as possible, each term will be defined and if possible an example of what that term represents will be given. BusCon is business consumption: the A matrix from before. HHCon is household consumption coefficients: this is a matrix of average propensities to consume by income class for each sector of production. GovCon is governmental consumption coefficients: this is governmental purchases of goods and services. BusValue is business value added coefficients: primarily profits and wages and salaries. HHValue is household-to-household value added coefficients: for example, households paying for babysitting services from other households. PayTran is coefficients for governmental payments and transfers to households, for example subsidized housing. BusTax is coefficients for business taxes, and HHTax is coefficients for household taxes. InterGov is coefficients inter-government transfers, for example federal grants to states for housing.

Investment coefficients are similarly divided into business investment (BusInv), household investment (HHInv), and government (GovInv). The difference between investment and consumption, is that investment only takes place with goods and services produced. Thus, the investment matrix, U_2 , is also a three by three matrix, but six of its entries are zero.

Two major elements remain of the model remain: exogenous final demand, F^* , and

$$U_2 = \begin{bmatrix} BusInv & HHInv & GovInv \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

generalized gross state product, X^* . Each of these elements is a vector with three components.

$$F^* = \begin{bmatrix} FinBus \\ FinHH \\ FinGov \end{bmatrix} \quad X^* = \begin{bmatrix} Output \\ HHGrossInc \\ GovOperEx \end{bmatrix}$$

FinBus is all other business sales and inventory changes such as exports. FinHH is other household income sources and net borrowing: this could be income from outside the region. FinGov is other government net cash sources. Output is X in the simple model. HHGrossInc is gross household income. GovOperEx is government operating expenditures.

Using the same technique as was used to create the production coefficient matrix and the investment coefficient matrix,¹ dividing U_1 and U_2 by \hat{X}^* creates a consumption coefficient matrix, R^* , and an investment coefficient matrix, B^* :

$$R^* = U_1 \hat{X}^{*-1} \quad B^* = U_2 \hat{X}^{*-1}$$

so that consumption and investment can be described in terms of coefficient matrixes:

$$R^* X^* = U_1 \quad B^* X^* = U_2$$

Equation (17) changes and can be solved for the matrix of multipliers:

$$(19) \quad \begin{aligned} X^* &= R^* \cdot X^* + B^* \cdot X^* + F^* \\ \Delta X^* &= (I - R^* - B^*)^{-1} \Delta F^* \end{aligned}$$

where $(I - R^* - B^*)^{-1}$ is the matrix of multipliers based on this social accounting framework.

¹ A vector turned into a diagonal matrix will have a hat over it.

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \rightarrow \quad \hat{Y} = \begin{bmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{33} \end{bmatrix}$$

REFERENCES

- Miller, Ronald E. and Peter D. Blair. *Input-Output Analysis: Foundations and Extensions*. Englewood Cliffs, New Jersey: Prentice-Hall: 1985.
- Pleeter, Saul. "Methodologies of Economic Impact Analysis: An Overview," in *Economic Impact Analysis: Methodology and Applications*. Ed. by Saul Pleeter. New York: Martinus Nijhoff Publishing, 1980.